2-2/IT

Code No. : 5277

FACULTY OF INFORMATICS

B.E. 2/4 (IT) II Semester (Suppl.) Examination, January 2012

PROBABILITY AND RANDOM PROCESS

Time: 3 Hours] [Max. Marks: 75

Note : Answer **all** questions from Part **A**. Answer **any five** questions from Part **B**.

PART – A (25 Marks)

1. Define the Axiomatic Definition of probability. 2
2. If A and B are independent events, prove that $\overline{A}$ and $\overline{B}$ are also independent. 2
3. A fair dice is rolled 5 times. Find the probability that 1 shows twice, 3 shows twice and 6 shows once. 3
4. State the properties of probability distribution function. 2
5. Find mean and variance of Poisson distribution. 3
6. State any three properties of characteristics function. 3
7. Write any three properties of auto correlation function. 3
8. Define Weiner-Kinchine theorem. 3
9. What is the difference between Random variable and Random process. 2
10. Define White-Noise. 2

PART – B (5×10=50 Marks)

11. a) Three switch S_1, S_2, and S_3 connected in parallel operate independently and each switch remains closed with probability P.
    b) Find the probability of receiving an input signal at the output.
    c) Find the probability that switch S_1 is open given that an input signal is received at the output. 10

12. a) Given the RV X with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the pdf of $Y = 8x^3$. 5

b) The joint pdf (X, Y) is given by $f(x, y) = 24xy$, $x > 0$, $y > 0$, $x + y \leq 1$, and $f(x, y) = 0$, elsewhere. Find the conditional mean and variance of y given x. 5

13. The joint probability distribution of a two-dimensional discrete RV (X, Y) is given below : 10

Y	X					
	0	1	2	3	4	5
0	0	0.01	0.03	0.05	0.07	0.09
1	0.01	0.02	0.04	0.05	0.06	0.08
2	0.01	0.03	0.05	0.05	0.05	0.06
3	0.01	0.02	0.04	0.06	0.06	0.05

i) Find marginal density functions of X and Y.

ii) Find $P(X \le 1), P(Y \le 3), P(X \le 1, Y \le 3), P(X \le 1/ Y \le 3), P(Y \le 3/x \le 1)$ and $P(X+Y \le 4)$.

14. a) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-a\tau^2}$. 5

b) Define semi-random telegraph signal process and random telegraph signal process and prove that semi-random telegraph signal process is evolutionary. 5

15. Consider a white Gaussian noise of zero mean and power spectral density No/2 applied to a low-pass RC filter whose transfer function is $H(f) = \frac{1}{1 + i2\pi fRC}$. Find the auto correlation function of the output random process. 10

16. If x(t) = 5 cos (10t + Φ) and y(t) = 20 sin (10t + ϕ) where Φ is a r.v. Uniformly distributed in (0, 2π), prove that the processes X(t) and Y(t) are jointly wide-stationary process. 10

17. a) The autocorrelation function for a stationary process is $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$. Find the mean and Variance of the process $X(\tau)$. 5

b) Given a stationary random process x(t) = 10 cos $[100t + \theta]$ where θ is a r.v. with a uniform probability distribution in the interval $[-\pi, \pi]$. Show that X(t) is ergodic in the autocorrelation function. 5

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