Code No. 9074 / M

FACULTY OF INFORMATICS

B.E. 2/4 (I.T) II – Semester (Main) Examination, May / June 2015

Subject: Probability and Random Processes

Time: 3 Hours **Max. Marks: 75**

Note: Answer all questions from Part A. Answer any five questions from Part B.

PART – A (25 Marks)

1 in a certain coin tossing experiment, let h and t be the outcomes denoted by space s = {h,t}. If the coin is tossed three times to get various outcomes such as hhh, hht, … Find the probability that exactly two h's appear in the outcome. 2

2 If A and M are events, and if *P*(A/M) is conditional probability of event A assuming M. Explain if the following statements are true:

i) $M \subset A$ then *P*(A/M) = 1

ii) $A \subset M$ then *P*(A)/*P*(M) ≥ *P*(A) 3

3 If random variable x takes the values 1 and 0 with probability p and q = 1-p. Show that the variance is equal to pq. 2

4 Plot the distribution function F(x) and density function f(x) for a fair die experiment with random variable x. 3

5 If x is a random variable with exponential distribution with $\lambda > 0$.
Given $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$. Find mean and variance. 2

6 If x and y are zero mean independent Gaussian random variables with common variance σ^2. Let $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$, where $|\theta| < \pi$. Find their joint density function. 3

7 Let $f_{xy}(x,y) = \begin{cases} 1 & 0 < |y| < x < 1 \\ 0 & \text{Otherwise} \end{cases}$ 3

Determine E{x|y} and E{y|x}.

8 Define stationary process. Write necessary and sufficient conditions for stationarity. 2

9 Write the mean value for white noise process and WSS process. 2

10 Find $R(\tau)$ if $s(w) = \dfrac{1}{(4 + w^2)^2}$. 3

PART – B (5x10 = 50 Marks)

11 State and prove Bernoulli's theorem. 10

12 A pair of dice is rolled on every play and the player wins at once if the total for the first throw is 7 or 11, losses if 2, 3 or 12 are rolled. Any other throw is called a "carry over". If the first throw is a carry over, then the player throws the dice repeatedly until he wins by throwing the same carry over again, or loses by throwing 7. What is the probability of winning the game? 10

13 a) Write the properties of distribution function. 6

b) A fair die is rolled five times. Find the probability that one shows twice, three shows twice, and six shows once. 4

14 a) A fair coin is tossed 10 times and x equals the number of heads.
Find i) $F_x(x)$ ii) $F_y(y)$ if $y = (x-3)^2$. 5

b) Define moments, m_n and characteristic functions $\phi_x(w)$ show that $\sigma^2 = m_2 - m_1^2 \geq 0$. 5

15 a) x and y are independent uniformly distributed random variables in (0,1).
Let w = max(x, y) and z = min (x, y). Find the p.d.f. of i) r = w-2 ii) s = w+z. 5

b) Find s(w) if i) $R(\tau) = e^{-\alpha\tau^2}$ ii) $R(\tau) = e^{-\alpha\tau^2 \cos w_o^{\tau}}$. 5

16 a) Define stochastic process and write second-order properties of stochastic process. 4

b) Define Mean-Ergodic process and show x(t) = a coswt + b sinwt + c is mean-ergodic, a and b are random variables with zero mean and equal variance. 6

17 a) If x(t) is a normal stationary process with zero mean, than the autocorrelation of the output of a hard 15 moter equals $R_y(\tau) = \frac{2}{\pi}$ arc sin $\frac{R_x(\tau)}{R_x(0)}$. 5

b) Show that the average power of a stationary process is independent of time and is equal to R(0). 5

****