Time and Work

1. Work from Days

If a person X can do a piece of work in n days, then X's 1 day's work = 1/ n

2. Days from Work

If X's 1 day's work = 1 / n, then X can finish the work in n days.

3. Ratio

If person X is 3 times as good a workman as Y,
Ratio of work done by X and Y = 3: 1.
Ratio of times taken by X and Y to finish a work = 1: 3.

Problems with solutions

1. A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day?

Solution

A's 2 day's work = $\left(\frac{1}{20} \text{ x } 2\right) = \frac{1}{10}$.

(A + B + C)'s 1 day's work = $\left(\frac{1}{20} + \frac{1}{30} + \frac{1}{60}\right) = \frac{6}{60} = \frac{1}{10}$.

Work done in 3 days = $\left(\frac{1}{10} + \frac{1}{10}\right) = \frac{1}{5}$.

Now, $\frac{1}{5}$ work is done in 3 days.

$\therefore$ Whole work will be done in (3 x 5) = 15 days.

2. A is thrice as good as workman as B and therefore is able to finish a job in 60 days less than B. Working together, they can do it in:

Solution

Ratio of times taken by A and B = 1 : 3.

The time difference is (3 - 1) 2 days while B take 3 days and A takes 1 day.

If difference of time is 2 days, B takes 3 days.

If difference of time is 60 days, B takes $\left(\frac{3}{2} \text{ x } 60\right)$ = 90 days.

So, A takes 30 days to do the work.

A's 1 day's work = $\frac{1}{30}$

B's 1 day's work = $\frac{1}{90}$

(A + B)'s 1 day's work = $\frac{1}{30} + \frac{1}{90} = \frac{4}{90} = \frac{2}{45}$

$\therefore$ A and B together can do the work in $\frac{45}{2} = 22\frac{1}{2}$ days.

3.
If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be:
Solution
Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.
Then, $6x + 8y = \frac{1}{10}$ and $26x + 48y = \frac{1}{2}$.
Solving these two equations, we get : $x = \frac{1}{100}$ and $y = \frac{1}{200}$.
(15 men + 20 boy)'s 1 day's work = $\frac{15}{100} + \frac{20}{200} = \frac{1}{4}$.

$\therefore$ 15 men and 20 boys can do the work in 4 days.

4. A can finish a work in 18 days and B can do the same work in 15 days. B worked for 10 days and left the job. In how many days, A alone can finish the remaining work?
Solution
B's 10 day's work = $\frac{1}{15} \times 10 = \frac{2}{3}$.
Remaining work = $1 - \frac{2}{3} = \frac{1}{3}$.
Now, $\frac{1}{18}$ work is done by A in 1 day.
$\therefore$ $\frac{1}{3}$ work is done by A in $\left(18 \times \frac{1}{3}\right)$ = 6 days.

5. A and B can together finish a work 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the work?
Solution
(A + B)'s 20 day's work = $\frac{1}{30} \times 20 = \frac{2}{3}$.
Remaining work = $\left(1 - \frac{2}{3}\right) = \frac{1}{3}$.
Now, $\frac{1}{3}$ work is done by A in 20 days.

Therefore, the whole work will be done by A in (20 x 3) = 60 days.