SURDS AND INDICES

S. No	Laws of Indices Formula
1	$X^{m} * X^{n} = X^{m+n}$
	$X^{m} / X^{n} = X^{m-n}$
	$(X^{m})^{n} = X^{m*n}$
	$X^{mn} = X^{m} * X^{n}$
	$X^{0}=1$

SURDS

Let x be rational number and n be a positive integer such that $x^{(1/n)} = x$.
Here x is called a surd of order n.

S. No	Laws of Surds Formula
1	$(a / b)^{1/n} = a / b$
	$((a)^{1/n})^{1/m} = (a)^{1/m\,n}$

Problems with solutions

1. If $5^{a} = 3125$, then the value of $5^{(a-3)}$ is:

Solution

$5^{a} = 3125 \quad \Leftrightarrow \quad 5^{a} = 5^{5}$

a = 5.

$5^{(a-3)} = 5^{(5-3)} = 5^{2} = 25$

2. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{?}$

Solution

Let $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{x}$.

Then, $\dfrac{(5^{2})^{7.5} \times (5)^{2.5}}{(5^{3})^{1.5}} = 5^{x}$

$\dfrac{5^{(2 \times 7.5)} \times 5^{2.5}}{5^{(3 \times 1.5)}} = 5x$

$\dfrac{5^{15} \times 5^{2.5}}{5^{4.5}} = 5^{x}$

$5^{x} = 5^{(15 + 2.5 - 4.5)}$

$5^{x} = 5^{13}$

x = 13.

3. $(0.04)^{-1.5}$ =?

Solution

$(0.04)^{-1.5} = \left(\frac{4}{100}\right)^{-1.5}$

$= \left(\frac{1}{25}\right)^{-(3/2)}$

$= (25)^{(3/2)}$

$= (5^2)^{(3/2)}$

$= (5)^{2 \times (3/2)}$

$= 5^3$

$= 125.$

4. If m and n are whole numbers such that $m^n = 121$, the value of $(m - 1)^{n + 1}$ is:

Solution

We know that $11^2 = 121$.

Putting m = 11 and n = 2, we get:

$(m - 1)^{n + 1} = (11 - 1)^{(2 + 1)} = 10^3 = 1000.$

5. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to:

Solution

$x^z = y^2$

$10^{(0.48z)} = 10^{(2 \times 0.70)} = 10^{1.40}$

$0.48z = 1.40$

$z = \frac{140}{48} = \frac{35}{12} = 2.9$ (approx.)