

RATIO AND PROPORTION

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction and we write it as $a : b$.

In the ratio $a : b$, we call a as the first term or antecedent and b, the second term or consequent.

Eg. The ratio 5: 9 represents $5/9$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

2. Proportion

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a : b :: c : d$ ($b \times c = a \times d$).

3. Fourth Proportional

If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b.

Mean Proportional:

Mean proportional between a and b is \sqrt{ab} .

4. Comparison of Ratios

We say that $(a : b) > (c : d) < \dots > a/b > c/d$

5. Compounded Ratio:

6. The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

7. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(\sqrt[3]{a} : \sqrt[3]{b})$.

If $a=c$, then

$$a + b = c + d$$

. [componendo and dividendo]

$$b d a - b c - d$$

8. Variations:

We say that x is directly proportional to y, if $x = ky$ for some constant k and we

Write, $x \propto y$.

We say that x is inversely proportional to y, if $xy = k$ for some constant k and

We write, x inversely proportional $1/y$

Problems with solutions

1. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

Solution

Let the third number be x .

$$\text{Then, first number} = 120\% \text{ of } x = \frac{120x}{100} = \frac{6x}{5}$$

$$\text{Second number} = 150\% \text{ of } x = \frac{150x}{100} = \frac{3x}{2}$$

$$\text{Ratio of first two numbers} = \left(\frac{6x}{5} : \frac{3x}{2} \right) = 12x : 15x = 4 : 5.$$

2. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

Solution

Let the shares of A, B, C and D be Rs. $5x$, Rs. $2x$, Rs. $4x$ and Rs. $3x$ respectively.

$$\text{Then, } 4x - 3x = 1000$$

$$x = 1000.$$

$$\text{B's share} = \text{Rs. } 2x = \text{Rs. } (2 \times 1000) = \text{Rs. } 2000.$$

3. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

Solution

Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$).

$$\left(\frac{140}{100} \times 5x \right), \left(\frac{150}{100} \times 7x \right) \text{ and } \left(\frac{175}{100} \times 8x \right)$$

$$7x, \frac{21x}{2} \text{ and } 14x.$$

$$\therefore \text{The required ratio} = 7x : \frac{21x}{2} : 14x$$

$$14x : 21x : 28x$$

$$2 : 3 : 4.$$

4. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

Solution

Originally, let the number of boys and girls in the college be $7x$ and $8x$ respectively. Their increased number is (120% of $7x$) and (110% of $8x$).

$$\left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$\frac{42x}{5} \text{ and } \frac{44x}{5}$$

$$\therefore \text{ The required ratio} = \left(\frac{42x}{5} : \frac{44x}{5}\right) = 21 : 22.$$

5. If $0.75 : x :: 5 : 8$, then x is equal to:

Solution

$$(x \times 5) = (0.75 \times 8) \Rightarrow x = 6/5 = 1.20$$