PIPES AND CISTERN

1. Inlet:

A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet:

A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

2. If a pipe can fill a tank in x hours, then: part filled in 1 hour =1/x

3. If a pipe can empty a tank in y hours, then part emptied in 1 hour =1/y

4. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (Where y > x), then on opening both the pipes, then the net part filled in 1 hour =1/x-1/y

5. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (Where x > y), then on opening both the pipes, then the net part emptied in 1 hour = (1 / y – 1 / x)

Problems with solutions

1. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:

Solution

Suppose, first pipe alone takes x hours to fill the tank .

Then, second and third pipes will take (x -5) and (x - 9) hours respectively to fill the tank.

$$\frac{1}{x} + \frac{1}{(x - 5)} = \frac{1}{(x - 9)}$$

$$\frac{x - 5 + x}{x(x - 5)} = \frac{1}{(x - 9)}$$

$$(2x - 5)(x - 9) = x(x - 5)$$

$$x^2 - 18x + 45 = 0$$

$$(x - 15)(x - 3) = 0$$

$x = 15$. [Neglecting $x = 3$]

2. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:

Solution

Work done by the waste pipe in 1 minute = $\frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right)$

$= \left(\frac{1}{15} - \frac{11}{120}\right)$

$= -\frac{1}{40}$. [-ve sign means emptying]

$\therefore$ Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3 x 40) gallons = 120 gallons.

3. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

Solution

Suppose pipe A alone takes x hours to fill the tank.

Then, pipes B and C will take $\frac{x}{2}$ and $\frac{x}{4}$ hours respectively to fill the tank.

$\therefore \quad \frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$

$\frac{7}{x} = \frac{1}{5}$

x = 35 hrs.

4. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

Solution

Part filled by A in 1 min = $\frac{1}{20}$.

Part filled by B in 1 min = $\frac{1}{30}$.

Part filled by (A + B) in 1 min = $\left(\frac{1}{20} + \frac{1}{30}\right) = \frac{1}{12}$.

$\therefore$ Both pipes can fill the tank in 12 minutes.

5. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half?

Solution

Part filled by (A + B) in 1 minute = $\left(\frac{1}{60} + \frac{1}{40}\right) = \frac{1}{24}$.

Suppose the tank is filled in x minutes.

Then, $\frac{x}{2}\left(\frac{1}{24} + \frac{1}{40}\right) = 1$

$\frac{x}{2} \times \frac{1}{15} = 1$

x = 30 min.