Permutation and Combination

1. Factorial Notation

If n is a positive integer. Factorial n denoted as n!. **n! = n (n - 1) (n - 2) ... 3.2.1.**

Examples

a. 0! = 1. b. 1! = (1 x 1) = 1. c. 2! = (2 x 1) = 2.	d. 3! = (3 x 2 x 1) = 6. e. 4! = (4 x 3 x 2 x 1) = 24.

2. Permutations / Arrangements

Each of several possible ways in which a set or number of things can be ordered or arranged Called permutations.

Examples

a. write permutations of letters a, b, c by taking 2 at a time are (ab, ba, ac, ca, bc, cb).
b. write permutations of letters a, b, c by taking all at a time are: (abc, acb, bac, bca, cab, cba)

3. Number of Permutations

Number of all permutations of n things, taken r at a time= nP_r
nPr = n (n - 1)(n - 2) ... (n - r + 1) = n! / (n - r)!

Examples

a. 5 P2 = 5 * 4 = 20.
b. 7 P4 = 7 * 6 * 5*4 = 840.

4. Combinations

The different selections possible from a collection of items are called combinations.

Examples:
If from 3 things A, B & C selected two things.

Combinations	**Permutations**
AB, BC and CA.	AB, BA, BC, CB, AC, CA

Note

Total Number of Combinations of n things, taken r at a time is given by

nCr = [n! / n (n - 1)(n - 2) (n - r)! = [n (n - 1)(n - 2) ... to r factors] / r!

Note	**Examples**
1. nCn = 1 2. nC0 = 1. 3. nCr = nC(n - r)	1. 11C4 = (11 x 10 x 9 x 8)/(4 x 3 x 2 x 1) = 330. 2. 16C13 = 16 C (16 - 13) = 3. 16C3 = 16 x 15 x 14 /3! = 16 x 15 x 14 / 3 x 2 x 1 = = 560.

Problems with solutions

1. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

Solution

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LNDG (EAI).

Now, 5 (4 + 1 = 5) letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

∴ Required number of ways = (120 x 6) = 720.

2. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

Solution

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$= ({}^7C_3 \times {}^4C_2)$

$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right)$

= 210.

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves = 5!

= 5 x 4 x 3 x 2 x 1

= 120.

Required number of ways = (210 x 120) = 25200.

3. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Solution

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

∴ Required number of ways $= ({}^6C_1 \text{ x } {}^4C_3) + ({}^6C_2 \text{ x } {}^4C_2) + ({}^6C_3 \text{ x } {}^4C_1) + ({}^6C_4)$

$= ({}^6C_1 \text{ x } {}^4C_1) + ({}^6C_2 \text{ x } {}^4C_2) + ({}^6C_3 \text{ x } {}^4C_1) + ({}^6C_2)$

$= (6 \text{ x } 4) + \left(\frac{6 \text{ x } 5}{2 \text{ x } 1} \text{ x } \frac{4 \text{ x } 3}{2 \text{ x } 1}\right) + \left(\frac{6 \text{ x } 5 \text{ x } 4}{3 \text{ x } 2 \text{ x } 1} \text{ x } 4\right) + \left(\frac{6 \text{ x } 5}{2 \text{ x } 1}\right)$

$= (24 + 90 + 80 + 15)$

$= 209.$

4. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?

Solution

Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it.

The tens place can now be filled by any of the remaining 5 digits (2, 3, 6, 7, 9). So, there are 5 ways of filling the tens place.

The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it. Required number of numbers = (1 x 5 x 4) = 20.

5. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

Solution

We may have(1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

∴ Required number of ways $= ({}^3C_1 \text{ x } {}^6C_2) + ({}^3C_2 \text{ x } {}^6C_1) + ({}^3C_3)$

$= \left(3 \text{ x } \frac{6 \text{ x } 5}{2 \text{ x } 1}\right) + \left(\frac{3 \text{ x } 2}{2 \text{ x } 1} \text{ x } 6\right) + 1$

$= (45 + 18 + 1) = 64$