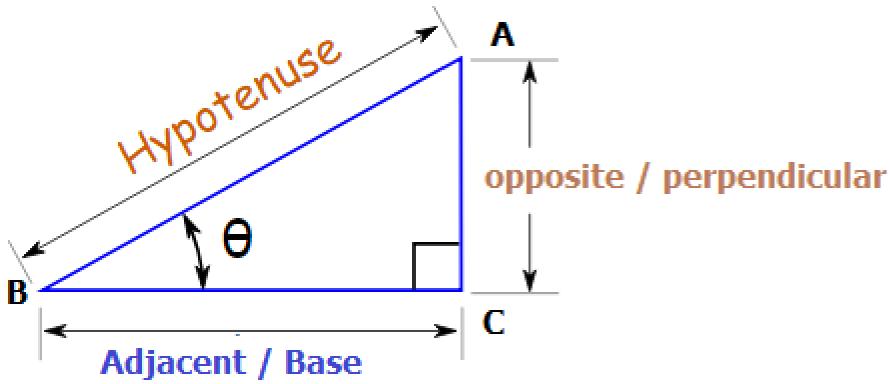
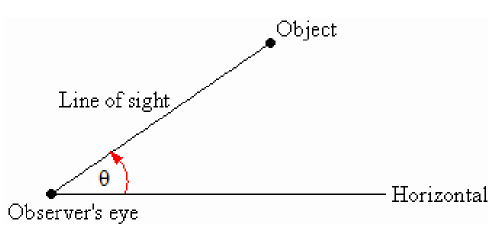
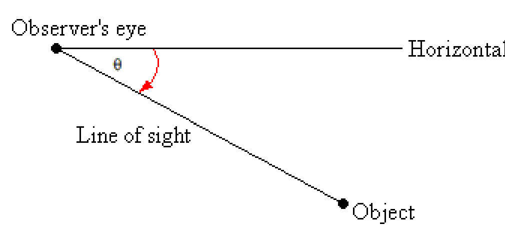
HEIGHT AND DISTANCE

Here ABC is a right angle Triangle



Formulas	**Trigonometric Identities**
Sin θ= Perpendicular/ Hypotenuse =AC/AB Cos θ= Adjacent/ Hypotenuse = BC/AB Tan θ= Perpendicular/ Adjacent =AC/BC	$\text{Sin}^2\,\theta + \cos^2\theta = 1$ $1 + \text{Tan}^2\,\theta = \sec^2\theta$ $1 + \text{Cot}^2\,\theta = \text{cosec}^2\,\theta$

θ	0	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	U

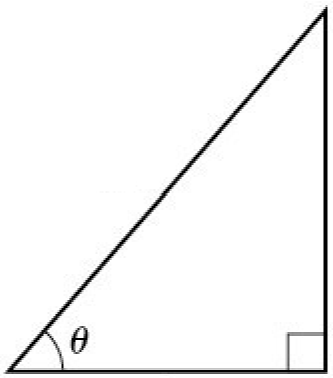
Angle of Elevation	**Angle of Depression**
It of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).	If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression.
	
The angle of elevation of the object from the observer is θ^0.	The angle of depression of the object from the observer is θ^0.

Problems with solutions

1. The angle of elevation of the sun, when the length of the shadow of a tree 3 times the height of the tree, is:

Solution

Let assume AB be the tree and AC be its shadow.



$\angle ACB = \theta$.

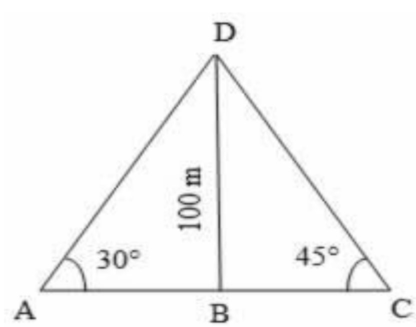
$\frac{AC}{AB} = 3 \rightarrow \cot\theta = 3$

$\theta = 30°$.

2. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:

Solution

Let AB be the lighthouse and C and D be the positions of the ships.



AB = 100 m, $\angle ACB = 30°$ and $\angle ADB = 45°$.

$\underline{AB}$ = tan 30° = $\underline{1}$ $\Rightarrow$ AC = AB x 3 = 1003 m.

AC 3

$\frac{AB}{AD}$ = tan 45° = 1 ⇒ AD = AB = 100 m.

CD = (AC + AD) = (1003 + 100) m

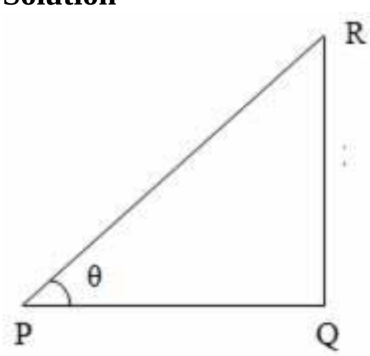
= 100(3 + 1)

= (100 x 2.73) m

= 273 m.

3. The angle of elevation of the sun, when the length of the shadow of a tree is equal to the height of the tree, is:

Solution



let QR represents the tree and PQ represents its shadow

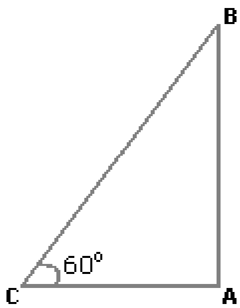
Here QR = PQ
Let ∠QPR = θ

tanθ=QRPQ=1 tanθ=QRPQ=1 (since QR = PQ)
θ = 45°

i.e., required angle of elevation = 45°

4. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:

Let AB = wall and BC = ladder.



∠ACB = 60° & AC = 4.6 m.

$$\frac{AC}{BC} = \cos 60° = \frac{1}{2}$$

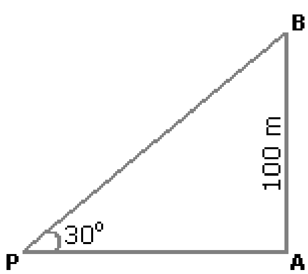
BC = 2 x AC

= (2 x 4.6) m

= 9.2 m.

5. From a point P on a level ground, the angle of elevation of the top tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is:

Let AB = tower.



∠APB = 30° and AB = 100 m.

$$\frac{AB}{AP} = \tan 30° = \frac{1}{3}$$

AP = (AB x 3) m

= 1003 m=(100 x 1.73) m

=173 m.