H.C.F AND L.C.M

1. Factors and Multiples

If number n1 / number n2 = exactly, Therefore n1 is a factor of n2 and n2 is a multiple of n1.

2. Highest Common Factor (H.C.F) / Greatest Common Measure (G.C.M) / Greatest Common Divisor (G.C.D)

The H.C.F. of 2 / more numbers is the greatest number that divides each of them exactly.

Two Methods of finding the H.C.F. of a given numbers

a. Factorization Method

Represent all numbers as a product of prime factors. The product of least powers of common prime factors gives H.C.F.

b. Division Method

To find the H.C.F. of 2 numbers, larger number divide by smaller number. Again divide divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is H.C.F.

Finding the H.C.F. of more than two numbers

If you want to find the H.C.F. of 3 numbers, then H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.

Similarly, the H.C.F. of more than three numbers may be obtained.

3. Least Common Multiple (L.C.M.)

The least number which is exactly divisible by each one of the given numbers is called as L.C.M. Two methods of finding the L.C.M. of a given set of numbers:

a. Factorization Method

Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.

b. Division Method (short-cut)

Arrange the all numbers in any order. Divide by a number which divided exactly at least 2 of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till numbers 2 of the numbers are divisible by the same number except 1. The product of the divisors & the undivided numbers is the required L.C.M. of the given numbers.

4. Product of two numbers = H.C.F * L.C.M

5. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

6. H.C.F. and L.C.M. of Fractions

a. H.C.F. =H.C.F. of Numerators / L.C.M. of Denominators

b. L.C.M. = L.C.M. of Numerators / H.C.F. of Denominators

7. H.C.F. and L.C.M. of Decimal Fractions:

In all the given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. / L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

8. Comparison of Fractions

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Problems with solutions

1. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

Solution

Required number = H.C.F. of (91 - 43), (183 - 91) and (183 - 43)

= H.C.F. of 48, 92 and 140 = 4

2. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

Solution

L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together $\frac{30}{2} + 1 = 16$ times.

3. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

Solution

Greatest number of 4-digits is 9999.

L.C.M. of 15, 25, 40 and 75 is 600.

On dividing 9999 by 600, the remainder is 399.

Required number (9999 - 399) = 9600.

4. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

Solution
Let numbers be 3x, 4x and 5x.
Their L.C.M. = 60x.
So, 60x = 2400 or x = 40.

The numbers are (3 x 40), (4 x 40) and (5 x 40).

Hence, required H.C.F. = 40.

5. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:

Solution

L.C.M. of 6, 9, 15 and 18 is 90.

Let required number be 90k + 4, which is multiple of 7.
Least value of k for which (90k + 4) is divisible by 7 is k = 4.

Required number = (90 x 4) + 4 = 364.