

Compound Interest

1. Let principal = p, Rate = R% per annum, Time = n years.

2. When interest is compounded annually:

$$\text{Amount} = p [1 + R/100]^n$$

3. When interest is compounded Half-Yearly:

$$\text{Amount} = p [1 + (R/2) / 100]^{2n}$$

4. When interest is compounded quarterly:

$$\text{Amount} = p [1 + (R/4) / 100]^{4n}$$

5. When interest is compounded annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = p [1 + R/100]^3 p [1 + (2/5) R / 100]$$

6. When Rates are different for different years, say R1%, R2%, R3% for 1st, 2nd and 3rd year respectively.

$$\text{Amount} = p [1 + R1/100] [1 + R2/100] [1 + R3/100]$$

7. Present worth of Rs. x due n years hence is given by:

$$\text{Present Worth} = x / (1 + R/100)^n$$

Problems with solutions

1. The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 4% per annum is Re. 1. The sum (in Rs.) is:

Solution

Let the sum be Rs. x. Then,

$$\text{C.I.} = \left[x \left(1 + \frac{4}{100} \right)^2 - x \right] = \left(\frac{676}{625}x - x \right) = \frac{51}{625}x.$$

$$\text{S.I.} = \left(\frac{x \times 4 \times 2}{100} \right) = \frac{2x}{25}.$$

$$\frac{51x}{625} - \frac{2x}{25} = 1$$

$$x = 625.$$

2. What is the difference between the compound interests on Rs. 5000 for $1\frac{1}{2}$ years at 4% per annum compounded yearly and half-yearly?

Solution

$$\text{C.I. when interest compounded yearly} = \text{Rs.} \left[5000 \times \left(1 + \frac{4}{100} \right) \times \left(1 + \frac{\frac{1}{2} \times 4}{100} \right) \right]$$

$$= \text{Rs.} \left(5000 \times \frac{26}{25} \times \frac{51}{50} \right)$$

$$= \text{Rs.} 5304.$$

C.I. when interest is compounded half-yearly

$$= \text{Rs.} \left[5000 \times \left(1 + \frac{2}{100} \right)^3 \right]$$

$$= \text{Rs.} \left(5000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \right)$$

$$= \text{Rs.} 5306.04$$

$$\text{Difference} = \text{Rs.} (5306.04 - 5304) = \text{Rs.} 2.04$$

3. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is:

Solution

$$\text{Amount} = \text{Rs.} (30000 + 4347) = \text{Rs.} 34347.$$

Let the time be n years.

$$\text{Then, } 30000 \left(1 + \frac{7}{100} \right)^n = 34347$$

$$\left(\frac{107}{100} \right)^n = \frac{34347}{30000} = \frac{11449}{10000} = \left(\frac{107}{100} \right)^2$$

$$n = 2 \text{ years.}$$

4. What will be the compound interest on a sum of Rs. 25,000 after 3 years at the rate of 12 p.c.p.a.?

4. What will be the compound interest on a sum of Rs. 25,000 after 3 years at the rate of 12 p.c.p.a.?

Solution

$$\text{Amount} = \text{Rs.} \left[25000 \times \left(1 + \frac{12}{100} \right)^3 \right]$$

$$= \text{Rs.} \left(25000 \times \frac{28}{25} \times \frac{28}{25} \times \frac{28}{25} \right)$$

$$= \text{Rs. } 35123.20$$

$$\therefore \text{C.I.} = \text{Rs. } (35123.20 - 25000) = \text{Rs. } 10123.20$$

5. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is:

Solution

$$\left. \begin{array}{l} \text{Amount of Rs. 100 for 1 year} \\ \text{when compounded half-yearly} \end{array} \right\} = \text{Rs. } \left[100 \times \left(1 + \frac{3}{100} \right)^2 \right] = \text{Rs. } 106.09$$

$$\text{Effective rate} = (106.09 - 100)\% = 6.09\%$$